# ARLINGTON <br> - COMMUNITY SCHOOL <br> Empowered and Inspired Today... Leading Our Community Tomorrow 

## $5^{\text {th }}$ Grade Math Pacing Guide 2018-2019

## Q1: Ready Mathematics Unit 1, Lessons 1-9: Number and Operations in Base Ten

Q2: Ready Mathematics Unit 2, Lessons 10-18: Number and Operations - Fractions
Q3: Ready Mathematics Units 3-4, Lessons 19-28: Operations and Algebraic Thinking; Measurement and Data; begin Geometry
Q4: Ready Mathematics Unit 5, Lessons 29-31: Geometry; Review; preview $6^{\text {th }}$ grade standards
$5^{\text {th }}$ grade fluency expectation: Fluently multiply multi-digit whole numbers (up to 3-digit by 4-digit factors) using appropriate strategies and algorithms.
Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

## Literacy Skills for Mathematical Proficiency

1. Use multiple reading strategies.
2. Understand and use correct mathematical vocabulary.
3. Discuss and articulate mathematical ideas.
4. Write mathematical arguments.

| $1^{\text {st }}$ Quarter | $\begin{gathered} \text { Instructional } \\ \text { Days } \end{gathered}$ | TN Standards | Vocabulary | Ready Math | Additional Resources |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Week 1: August 7-11 | 5 | 5.NBT.B.5: Fluently multiply multi-digit whole numbers (up to 3-digit by 4-digit factors) using appropriate strategies and algorithms. <br> (See Table 3 - Properties of Operations) | distributive property, factor, product, partial products | Lesson 5: <br> Multiply <br> Whole <br> Numbers | Study Island: 3e. Multiply Whole Numbers <br> TN Performance Coach Lesson 9 <br> NC Tasks <br> Khan Academy <br> Illustrative Mathematics <br> Engage NY <br> Learnzillion <br> Learning Farm |

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| Week 6: September 11 15 | 4 | 5.NBT.A.2: Explain patterns in the number of zeros of the product when multiplying a number by powers of 10 , and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10 . Use whole-number exponents to denote powers of 10 | exponent, power of ten, inverse operations, decimal | Lesson 2: <br> Understand Powers of Ten | Study Island: 3a. Powers of Ten TN Performance Coach Lesson 5 NC Tasks <br> Learnzillion <br> EngageNY <br> Khan Academy <br> Illustrative Mathematics <br> Learning Farm |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Week 7: September 18 22 | 5 | 5.NBT.B.7: Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between operations; assess the reasonableness of answers using estimation strategies. (Limit division problems so that either the dividend or divisor is a whole number) <br> (See Table 3 - Properties of Operations) | decimal, to estimate, place value, sum, difference | Lesson 7: Add and Subtract Decimals | Study Island: 3g. Add and Subtract <br> Decimals <br> ITN Performance Coach Lesson 11 <br> NC Tasks <br> Khan Academy <br> Illustrative Mathematics <br> EngageNY <br> Learnzillion <br> Learning Farm |
| Week 8: Sept. 25-29 | 5 | 5.NBT.B.7: Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between operations; assess the reasonableness of answers using estimation strategies. (Limit division problems so that either the dividend or divisor is a whole number) <br> (See Table 3 - Properties of Operations) | decimal, product, factor, place value, to estimate | Lesson 8: <br> Multiply <br> Decimals | Study Island: 3h. Multiply and Divide Decimals <br> TN Performance Coach Lesson 12 <br> NC Tasks <br> Khan Academy <br> Illustrative Mathematics <br> EngageNY <br> Learnzillion <br> Learning Farm |
| Week 9: October 2-6 | 5 | 5.NBT.B.7: Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between operations; assess the reasonableness of answers using estimation strategies. (Limit division problems so that either the dividend or divisor is a whole number) <br> (See Table 3 - Properties of Operations) | dividend, divisor, quotient, to estimate | Lesson 9: Divide Decimals (at this time, Ready includes dividing decimals by decimals) | Study Island: 3h. Multiply and Divide <br> Decimals <br> TN Performance Coach Lesson 13 <br> NC Tasks <br> Khan Academy <br> Illustrative Mathematics <br> EngageNY <br> Learnzillion <br> Learning Farm |
| End of $1^{\text {st }}$ Quarter |  | District Quarterly CFA |  |  |  |
| Fall Break October 9-13 |  |  |  |  |  |
| $2^{\text {nd }}$ Quarter | $\begin{aligned} & \text { Instructional } \\ & \text { Davs } \end{aligned}$ | TN Standards | Vocabulary | Ready Math | Additional Resources |
| Week 1: Oct. 16-20 | 5 | 5.NF.A.1: Add and subtract fractions with unlike denominators (including mixed numbers) by replacing | numerator, denominator, | Lesson 10: Add and Subtract | Study Island: 4a. Add and Subtract Fractions |

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|  |  |  | given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, $2 / 3+5 / 4=8 / 12+$ $15 / 12=23 / 12$. (In general, $a / b+c / d=(a d+b c) / b d$.) <br> (See Table 1 - Addition and Subtraction Situations for whole number situations that can be applied to fractions) | equivalent fractions, common denominator | Fractions | TN Performance Coach Lesson 14 NC Tasks <br> Illustrative Mathematics <br> EngageNY <br> Learnzillion <br> Khan Academy <br> Learning Farm |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Week 2: October 23-27 | 5 | 5.NF.A.2: Solve contextual problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators. Use benchmark <br> fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result $2 / 5+1 / 2=3 / 7$, by observing that $3 / 7<1 / 2$. <br> (See Table 1 - Addition and Subtraction Situations for whole number situations that can be applied to fractions) | common denominator, equivalent fractions, benchmark fraction | Lesson 11: Add and Subtract Fractions in Word Problems | Study Island: 4b. Real World Add and <br> Subtract Fractions <br> TN Performance Coach Lesson 15 <br> NC Tasks <br> Illustrative Mathematics <br> EngageNY <br> Learnzillion <br> Khan Academy <br> Learning Farm |
|  | Week 3: Oct. 30-Nov. 3 | 5 | 5.NF.B.3: Interpret a fraction as division of the numerator by the denominator $(a / b=a \div b)$. For example, $3 / 4=3$ divided by 4 , so when 3 wholes are shared equally among 4 people, each person has a share of size $3 / 4$. Solve contextual problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers by using visual fraction models or equations to represent the problem. For example, if 8 people want to share 49 sheets of construction paper equally, how many sheets will each person receive? Between what two whole numbers does your answer lie? <br> (See Table 2 - Multiplication and Division Situations for whole number situations that can be applied to fractions) | fraction, numerator, denominator, quotient | Lesson 12: <br> Fractions as Division | Study Island: 4f. Interpreting Fractions as Division <br> TN Performance Coach Lesson 16 <br> Xtramath, Moby Max, iReady, One <br> Drive Resources <br> NC Tasks <br> Illustrative Mathematics <br> EngageNY <br> Learnzillion <br> Khan Academy <br> Learning Farm |
|  | Week 4: November 6 10 | 5 | 5.NF.B.4: Apply and extend previous understandings of multiplication to multiply a fraction by a whole number or a fraction by a fraction. <br> 5.NF.B.4.a: Interpret the product of $(a / b) \times q$ as $a \times(q \div$ b) (partition the quantity $q$ into $b$ equal parts and then multiply by $a$ ). Interpret the product $a / b \times q$ as ( $a \times q$ ) $\div b$ (multiply $a$ times the quantity $q$ and then partition the product into $b$ equal parts). For example, use a visual fraction model or write a story context to show that $2 / 3 x$ 6 can be interpreted as $2 \times(6 \div 3)$ or $(2 \times 6) \div 3$. Do the | numerator, denominator, unit fraction, product, factor | Lesson 13: <br> Understand <br> Products of Fractions | Study Island: 4c. Multiplication with Fractions <br> TN Performance Coach Lesson 17 <br> NC Tasks <br> Illustrative Mathematics <br> EngageNY <br> Learnzillion <br> Khan Academy <br> Learning Farm |

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| Week 8: December 4-8 | 5 | 5.NF.B.7: Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. <br> 5.NF.B.7.a: Interpret division of a unit fraction by a nonzero whole number, and compute such quotients. For example, use visual models and the relationship between multiplication and division to explain that $(1 / 3) \div 4=$ $1 / 12$ because $(1 / 12) \times 4=1 / 3$. <br> 5.NF.B.7b: Interpret division of a whole number by a unit fraction, and compute such quotients. For example, use visual models and the relationship between multiplication and division to explain that $4 \div(1 / 5)=20$ because $20 \times(1 / 5)=4$. <br> (See Table 2 - Multiplication and Division Situations for whole number situations that can be applied to fractions) | unit fractions | Lesson 17: <br> Understand <br> Division with <br> Unit Fractions | Study Island: 4g. Division With <br> Fractions <br> TN Performance Coach Lesson 21 <br> NC Tasks <br> Illustrative Mathematics <br> EngageNY <br> Learnzillion <br> Khan Academy <br> Learning Farm |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Week 9: Dec. 11-15 | 5 | 5.NF.B.7c: Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share $1 / 2 \mathrm{lb}$. of chocolate equally? How many $1 / 3$-cup servings are in 2 cups of raisins? <br> (See Table 2 - Multiplication and Division Situations for whole number situations that can be applied to fractions) | unit fractions | Lesson 18: <br> Divide Unit <br> Fractions in Word Problems | Study Island: 4g. Division With <br> Fractions <br> TN Performance Coach Lesson 22 <br> NC Tasks <br> Illustrative Mathematics <br> EngageNY <br> Learnzillion <br> Khan Academy <br> Learning Farm |
| Week 10: Dec. 18-22 | 2.5 |  |  |  |  |
| End of $2^{\text {nd }}$ Quarter |  | District | rterly CFA |  |  |
| End of $1^{\text {st }}$ Semester |  |  |  |  |  |
|  |  | Winter Break Dec | ber 21 - Ja |  |  |
| $3^{\text {rd }}$ Quarter | Instructional Days | TN Standards | Vocabulary | Ready Math | Additional Resources |
| Week 1: January 1-5 | 2 | 5.0A.A.1: Use parentheses and/or brackets in numerical expressions, and evaluate expressions having these symbols using the conventional order (Order of Operations). <br> 5.OA.A.2: Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation "add 8 and 7 , then multiply by 2 " as $2 \times(8+7)$. Recognize that $3 \times(18,932+921)$ is three | evaluate, parentheses | Lesson 19: <br> Evaluate and Write Expressions | Study Island: 2a. Order of Operations; <br> 2b. Numerical Expressions <br> TN Performance Coach Lessons 1-2 <br> NC Tasks <br> Illustrative Mathematics <br> EngageNY OA.A. 1 <br> EngageNY OA.A. 2 <br> Learnzillion <br> Khan Academy |

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|  |  | times as large as $18,932+921$, without having to calculate the indicated sum or product. |  |  | $\begin{aligned} & \text { Learning Farm OA.A. } 1 \\ & \text { Learning Farm OA.A. } 2 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Week 2: January 8-12 | 5 | 5.OA.B.3: Generate two numerical patterns using two given rules. For example, given the rule "Add 3" and the starting number 0 , and given the rule "Add 6 " and the starting number 0 , generate terms in the resulting sequences. <br> 5.OA.B.3.a: Identify relationships between corresponding terms in two numerical patterns. For example, observe that the terms in one sequence are twice the corresponding terms in the other sequence. <br> 5.OA.B.3.b: Form ordered pairs consisting of corresponding terms from two numerical patterns, and graph the ordered pairs on a coordinate plane. | corresponding terms, ordered pair | Lesson 20: <br> Analyze <br> Patterns and <br> Relationships | Study Island: 2c. Number Patterns <br> TN Performance Coach Lesson 3 <br> NC Tasks <br> Illustrative Mathematics <br> EngageNY <br> Learnzillion <br> Khan Academy <br> Learning Farm |
| Week 3: January 15-19 | 4 | 5.MD.A.1: Convert customary and metric measurement units within a single system by expressing measurements of a larger unit in terms of a smaller unit. Use these conversions to solve multi-step real-world problems involving distances, intervals of time, liquid volumes, masses of objects, and money (including problems involving simple fractions or decimals). For example, 3.6 liters and 4.1 liters can be combined as 7.7 liters or 7700 milliliters. | convert, metric system, customary system | Lesson 21: <br> Convert <br> Measurement Units | Study Island: 5a. Units of Measure <br> TN Performance Coach Lesson 23 <br> NC Tasks <br> Illustrative Mathematics <br> EngageNY <br> Learnzillion <br> Khan Academy <br> Learning Farm |
| Week 4: January 22-26 | 5 | 5.MD.A.1: Convert customary and metric measurement units within a single system by expressing measurements of a larger unit in terms of a smaller unit. Use these conversions to solve multi-step real-world problems involving distances, intervals of time, liquid volumes, masses of objects, and money (including problems involving simple fractions or decimals). For example, 3.6 liters and 4.1 liters can be combined as 7.7 liters or 7700 milliliters. | metric system, customary system | Lesson 22: Solve Word Problems Involving Conversions | Study Island: 5a. Units of Measure <br> TN Performance Coach Lesson 23 <br> NC Tasks <br> Illustrative Mathematics <br> EngageNY <br> Learnzillion <br> Khan Academy <br> Learning Farm |
| Week 5: Jan. 29 - Feb. 2 | 5 | 5.MD.B.2: Make a line plot to display a data set of measurements in fractions of a unit ( $1 / 2,1 / 4,1 / 8$ ). Use operations on fractions for this grade to solve problems involving information presented in line plots. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally. | distribution, line plot, scale | Lesson 23: <br> Make Line Plots and Interpret Data | Study Island: 5b. Representing and Interpreting Data <br> TN Performance Coach Lesson 24 <br> NC Tasks <br> Illustrative Mathematics <br> EngageNY <br> Learnzillion <br> Khan Academy |


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| :---: | :---: | :---: | :---: | :---: | :---: |
| Week 6: February 5-9 | 5 | 5.MD.C.3: Recognize volume as an attribute of solid figures and understand concepts of volume measurement. <br> 5.MD.C.3a: Understand that a cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. <br> 5.MD.C.3b: Understand that a solid figure, which can be packed without gaps or overlaps using $n$ unit cubes is said to have a volume of $n$ cubic units. | plane figure, solid figure, volume, cubic unit, rectangular prism | Lesson 24: <br> Understand <br> Volume | Study Island: 5c. Volume - Unit Cubes <br> TN Performance Coach Lesson 25 <br> NC Tasks <br> Illustrative Mathematics <br> EngageNY <br> Learnzillion <br> Khan Academy <br> Learning Farm |
| Week 7: February 12 16 | 4 | 5.MD.C.4: Measure volumes by counting unit cubes, using cubic cm , cubic in, cubic ft , and improvised units. | volume, cubic unit, rectangular prism | Lesson 25: Find Volume Using Unit Cubes | Study Island: 5d. Volume with Unit Cubes <br> TN Performance Coach Lesson 26 <br> NC Tasks <br> Illustrative Mathematics <br> EngageNY <br> Learnzillion <br> Khan Academy <br> Learning Farm |
| Week 8: February 19 - <br> 23 | 4 | 5.MD.C.5: Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume of right rectangular prisms. <br> 5.MD.C.5a: Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent whole-number products of three factors as volumes, (e.g., to represent the associative property of multiplication). <br> 5.MD.C.5b: Know and apply the formulas $V=I \times w \times h$ and $V=B \times h$ (where $B$ represents the area of the base) for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems. | area, formula, volume, cubic unit | Lesson 26: Find Volume Using Formulas | Study Island: 5d. Volume with Unit <br> Cubes; 5e. Volume <br> TN Performance Coach Lesson 26 <br> NC Tasks <br> Illustrative Mathematics <br> EngageNY <br> Learnzillion <br> Khan Academy <br> Learning Farm |
| Week 9: Feb. 26 March 2 | 5 | 5.MD.C.5c: Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the nonoverlapping parts, applying this technique to solve real world problems. | area, square unit, volume, cubic unit | Lesson 27: Find <br> Volume of Composite Figures | Study Island: 5f. Volume of Composite <br> Figures <br> TN Performance Coach Lesson 27 <br> NC Tasks <br> Illustrative Mathematics |

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|  |  |  |  |  | EngageNY <br> Learnzillion <br> Khan Academy <br> Learning Farm |
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| Week 10: March 5-9 | 5 | 5.G.A. 1 Graph ordered pairs and label points using the first quadrant of the coordinate plane. Understand in the ordered pair that the first number indicates the horizontal distance traveled along the x -axis from the origin and the second number indicates the vertical distance traveled along the $y$-axis, with the convention that the names of the two axes and the coordinates correspond (e.g., $x$-axis and $x$-coordinate, $y$-axis and $y$ coordinate). | coordinate plane, $x$-axis, $y$ axis, ordered pair, $x$ coordinate, $y$ coordinate, origin | Lesson 28: Understand the Coordinate Plane | Study Island: 6a. Coordinate System <br> TN Performance Coach Lesson 28 <br> NC Tasks <br> Illustrative Mathematics <br> EngageNY <br> Learnzillion <br> Khan Academy <br> Learning Farm |
| End of $3^{\text {rd }}$ Quarter |  | District Quart | rly CFA |  |  |
|  |  | Spring Break M | 12-16 |  |  |
| $4^{\text {th }}$ Quarter | $\begin{gathered} \text { Instructional } \\ \text { Days } \end{gathered}$ | TN Standards | Vocabulary | Ready Math | Additional Resources |
| Week 1: March 19-23 | 5 | 5.G.A.2: Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation. | coordinate plane, ordered pair, origin | Lesson 29: Graph Points in the Coordinate Plane | Study Island: 6a. Coordinate System <br> TN Performance Coach Lesson 29 <br> NC Tasks <br> Illustrative Mathematics <br> EngageNY <br> Learnzillion <br> Khan Academy <br> Learning Farm |
| Week 2: March 26-30 | 4 | 5.G.B. 3 Classify two-dimensional figures in a hierarchy based on properties. Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles. | hierarchy, polygon, Venn diagram | Lesson 30: Classify TwoDimensional Figures | Study Island: 6b. 2-Dimensional <br> Figures <br> TN Performance Coach Lesson 30 <br> NC Tasks <br> Illustrative Mathematics <br> EngageNY <br> Learnzillion <br> Khan Academy <br> Learning Farm |
| Week 3: April 2-6 | 5 | 5.G.B. 3 Classify two-dimensional figures in a hierarchy based on properties. Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles. | convex polygon, concave polygon, attribute | Lesson 31: <br> Understand Properties of TwoDimensional Figures | Study Island: 6b. 2-Dimensional Figures <br> TN Performance Coach Lesson 30 <br> NC Tasks <br> Illustrative Mathematics <br> EngageNY <br> Learnzillion |

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|  |  |  | signs, the locations of the points are related by <br> reflections across one or both axes. <br> 6.NS.6c Find and position integers and other rational <br> numbers on a horizontal or vertical number line diagram; <br> find and position pairs of integers and other rational <br> numbers on a coordinate plane. | $\frac{\text { s.com/SortedBy }}{\text { Grade.php?Sort }}$ <br> Week 10: May 21-25 | 3.5 | ens6c |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Table 1 Common addition and subtraction situations

|  | Result Unknown | Change Unknown | Start Unknown |
| :---: | :---: | :---: | :---: |
| Add to | Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2+3=?$ | Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2+?=5$ | Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $?+3=5$ <br> One-Step Problem $\left(2^{\text {nd }}\right)$ |
| Take from | Five apples were on the table. I ate two apples. How many apples are on the table now? $5-2=?$ | Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5-?=3$ | Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $?-2=3$ <br> One-Step Problem <br> $\left(2^{\text {nd }}\right)$ |
| Put Together/ Take Apart ${ }^{3}$ | Total Unknown | Addend Unknown | Both Addends Unknown ${ }^{2}$ |
|  | Three red apples and two green apples are on the table. How many apples are on the table? $3+2=$ ? | Five apples are on the table. Three are red and the rest are green. How many apples are green? $3+?=5,5-3=?$ | Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $\begin{aligned} & 5=0+5,5=5+0 \\ & 5=1+4,5=4+1 \\ & 5=2+3,5=3+2 \end{aligned}$ |
| Compare ${ }^{4}$ | Difference Unknown | Bigger Unknown | Smaller Unknown |
|  | ("How many more?" version): <br> Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? | (Version with "more"): <br> Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? <br> One-Step Problem <br> (1) | (Version with "more"): <br> Julie has 3 more apples than Lucy. Julie has five apples. How many apples does Lucy have? $5-3=? \quad ?+3=5$ <br> One-Step Problem |
|  | ("How many fewer?" version): <br> Lucy has two apples. Julie has five apples. <br> How many fewer apples does Lucy have than Julie? $2+?=5,5-2=?$ <br> (17) | (Version with "fewer"): <br> Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2+3=?, 3+2=?$ <br> One-Step Problem $\left(2^{\mathrm{nd}}\right)$ | (Version with "fewer"): <br> Lucy has three fewer apples than Julie. Julie has five apples. How many apples does Lucy have? <br> One-Step Problem |

K: Problem types to be mastered by the end of the Kindergarten year.
1st: Problem types to be mastered by the end of the First Grade year, including problem types from the previous year. However, First Grade students should have experiences with all 12 problem types.
2nd: Problem types to be mastered by the end of the Second Grade year, including problem types from the previous years.

Table 2 Common multiplication and division situations ${ }^{1}$

\begin{tabular}{|c|c|c|c|}
\hline \& Unknown Product

$3 \times 6=$ ? \& | Group Size Unknown |
| :--- |
| ("How many in each group?" |
| Division) $3 \times ?=18 \text {, and } 18 \div 3=\text { ? }$ | \& Number of Groups Unknown ("How many groups?" Division)

$$
? \times 6=18 \text {, and } 18 \div 6=?
$$ <br>

\hline Equal Groups \& | There are 3 bags with 6 plums in each bag. How many plums are there in all? |
| :--- |
| Measurement example. You need 3 lengths of string, each 6 inches long. How much string will you need altogether? | \& | If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? |
| :--- |
| Measurement example. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be? | \& | If 18 plums are to be packed 6 to $a$ bag, then how many bags are needed? |
| :--- |
| Measurement example. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have? | <br>


\hline Arrays, ${ }^{2}$ Area ${ }^{3}$ \& | There are 3 rows of apples with 6 apples in each row. How many apples are there? |
| :--- |
| Area example. What is the area of a 3 cm by 6 cm rectangle? | \& | If 18 apples are arranged into 3 equal rows, how many apples will be in each row? |
| :--- |
| Area example. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it? | \& | If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? |
| :--- |
| Area example. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it? | <br>

\hline \& A blue hat costs $\$ 6$. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? \& A red hat costs $\$ 18$ and that is 3 times as much as a blue hat costs. How much does a blue hat cost? \& A red hat costs $\$ 18$ and a blue hat costs $\$ 6$. How many times as much does the red hat cost as the blue hat? <br>
\hline Compare \& Measurement example. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long? \& Measurement example. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first? \& Measurement example. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first? <br>
\hline General \& $a \times b=$ ? \& $a \times ?=p$, and $p \div a=$ ? \& $? \times b=p$, and $p \div b=$ ? <br>
\hline
\end{tabular}

[^0]
## Table 3 The properties of operations

Here $a, b$ and $c$ stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

| Associative property of addition $(a+b)+c=a+(b+c)$ <br> Commutative property of addition $a+b=b+a$ <br> Additive identity property of 0 $a+0=0+a=a$ <br> Associative property of multiplication $(a \times b) \times c=a \times(b \times c)$ <br> Commutative property of multiplication $a \times b=b \times a$ <br> Multiplicative identity property of 1 $a \times 1=1 \times a=a$ <br> Distributive property of multiplication over addition $a \times(b+c)=a \times b+a \times c$ |
| :--- | :---: |


[^0]:    'Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).
    ${ }^{2}$ The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.
    ${ }^{3}$ Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

